# A Synthesis of Mathematics Writing: Assessments, Interventions, and Surveys

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**Abstract:** Mathematics standards in the United States describe communication as an essential part of mathematics. One outlet for communication is writing. To understand the mathematics writing of students, we conducted a synthesis to evaluate empirical research about mathematics writing. We identified 29 studies that included a mathematics-writing assessment, intervention, or survey for students in 1st through 12th grade. All studies were published between 1991 and 2015. The majority of assessments required students to write explanations to mathematical problems, and fewer than half scored student responses according to a rubric. Approximately half of the interventions involved the use of mathematics journals as an outlet for mathematics writing. Few intervention studies provided explicit direction on how to write in mathematics, and a small number of investigations provided statistical evidence of intervention efficacy. From the surveys, the majority of students expressed enjoyment when writing in mathematics settings but teachers reported using mathematics writing rarely. Across studies, findings indicate mathematics writing is used for a variety of purposes, but the quality of the studies is variable and more empirical research is needed.

**Keywords:** writing, mathematics, mathematics writing, mathematical communication

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## 1. Introduction

Mathematical competence is necessary for success throughout school and adulthood. For example, mathematics in kindergarten is the single best predictor of academic achievement at the end of elementary school (Claessens, Duncan, & Engel, 2008), and mathematics in the late elementary grades and middle school is predictive of academic success in high school (Lee, 2012; Wilkins & Ma, 2002). In turn, better high school mathematical performance leads to improved post-secondary options and adulthood outcomes (Dougherty, 2003). As mathematics expectations in the United States have increased with the introduction of standards focused on advanced mathematical content and practice (Cobb & Jackson, 2011), there has been increased attention given to using multiple methods for demonstrating mathematical competence. In mathematics classrooms, not only are students required to demonstrate mathematical concepts and procedures, but students are also expected to use writing to communicate mathematically. This is true in the United States and across the globe. In this manuscript, we synthesized research related to mathematics writing from 29 studies conducted in 10 countries. The goal of this synthesis was to determine how researchers and educators teach and assess mathematics writing, as well as how teachers and students feel about mathematics writing.

Before proceeding, we comment on terminology in this manuscript. We broadly defined mathematics writing as any writing related to mathematics. As described by Bossé and Faulconer (2008), students may write about mathematics (e.g., write about mathematicians) or write in mathematics (e.g., write explanations of mathematical concepts). To understand the extent to which students do either, we used a broad definition of *mathematics* (i.e., any content taught in a mathematics course) and *writing* (i.e., using written words typed or handwritten). Note that we did not define mathematics writing as writing numerals (without words) or writing equations or expressions (without words). We defined assessment as any mathematics-writing activity in which students wrote about or in mathematics and the students' writing was analyzed or scored. We classified intervention as any classroom activity introduced or led by the teacher in which students wrote about or in mathematics or wrote to learn mathematics. Our definition of survey included questions asked about mathematics-writing practices or experiences.

### 1.1 Communication Using Mathematics Writing

Over the last two decades in the United States, mathematics writing has become more prevalent as a means for communicating mathematical ideas. The National Council of Teachers of Mathematics (NCTM, 2000) described communication as "an essential part of mathematics" (p. 60), and listed communication as one of the five process standards for effective mathematical instruction. NCTM expressed that written communication in mathematics is not often the focus of mathematics education; therefore, students require instruction on using the language of mathematics to express ideas, both oral and written. Additionally, the Common Core's standards for mathematical practice (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) suggested students be able to construct viable arguments, critique the mathematical reasoning of others, explain how to solve problems, use clear definitions and vocabulary, and communicate precisely to others.

Many students participate in mathematical discourse to engage in mathematical communication (Ben-Yehuda, Lavy, Linchevski, & Sfard, 2005; Jung & Reifel, 2011), and mathematics writing is an additional method for students to communicate about mathematics (Bicer, Capraro, & Capraro, 2014). The difficulty with requiring students to communicate mathematically via writing is that, according to The Nation's Report Card, only 27% of eighth-grade students in the United States scored at or above proficient on the National Assessment of Educational Progress writing assessment during the most recent administration (National Center for Education Statistics, 2012), with 20% of students scoring "below basic." The results are similar for 12th grade. Grade 4 was not assessed in writing in 2011, but only 33% of fourth graders performed at the proficient level in 2009 (National Center for Educational Statistics). These results indicate that the majority of students are not meeting grade level demands in writing. It is likely that students experiencing difficulty with writing also struggle with mathematics writing (Hebert & Powell, 2016).

### 1.2 Gauging Mathematics Knowledge with Mathematics-Writing Assessments

Despite the low writing performance of school-aged students, students are asked to write mathematically to answer mathematical questions and demonstrate mathematical competence on high-stakes assessments and performance tasks. For example, students may be prompted to "use math and words to explain the work," "show how you got your answer…using words," "use words to explain why or why not," "explain your reasoning using words, numbers, and pictures," or "identify the…mistake and explain what he should do to correct it" (www.parcconline.org; www.smarterbalanced.org). On several performance tasks, students are pressed to move beyond explaining work with mathematics writing: students may be asked to use mathematics writing to compare two amounts, decide whether amounts can be equal, and decide which purchase option would be cheaper.

A challenge related to assessing students' mathematical knowledge through writing involves the inherent assumptions that must be made. Powell and Hebert (2016) identified three implied assumptions of mathematics-writing assessments. One, students possess the writing skill to respond adequately to a mathematics-writing prompt. Two, students transfer general writing knowledge to mathematics writing. Three, mathematics writing demonstrates mathematical competence. These assumptions, however, may not be valid (see Powell & Hebert, 2016, for a full description of the theory), leading to questions about what, exactly, is being measured.

Related to this issue, there are few standards for judging the quality of mathematics-writing responses. For mathematics-writing items on the assessments related to the Common Core in the United States (i.e., PARCC and Smarter Balanced), mathematics writing is scored against a rubric specifically designed to assess both writing organization and mathematical communication. These rubrics often include graduated score patterns where students earn more points for more in-depth mathematics writing. One potential limitation to these rubrics is that it is unclear whether the rubrics help gauge mathematical knowledge, writing skill, or both. Most likely, mathematics-writing assessments measure a combination of mathematics and writing skills, as demonstrated by recent empirical research (Powell & Hebert, 2016). Therefore, it may be important to understand whether mathematical knowledge and writing skill are weighted appropriately in an overall score or whether writing skill disproportionately impacts scores.

### 1.3 Improving Mathematical Knowledge with Mathematics-Writing Instruction

In addition to being a means for communicating about mathematics and other knowledge, writing can be used as a learning tool to improve reading and content area learning outcomes (Graham & Hebert, 2011; Klein & Kirkpatrick, 2010). Huang, Normandia, and Greer (2005) suggested that mathematical communication is an integral part of building an understanding about mathematics. In partial support of this, in a meta-analysis of writing studies conducted through 1999, content area writing interventions (including mathematics-writing interventions), had statistically significant and positive effects on school achievement outcomes (Bangert-Drowns, Hurley, & Wilkinson, 2004). Data were not disaggregated, however, for mathematics writing, and none of the studies related to mathematics was peer-reviewed.

To provide effective mathematical instruction, it is necessary to understand the quantity of mathematics-writing instruction teachers implement in classrooms, what type of instruction is provided on mathematics writing, and how comfortable teachers are with providing such instruction on mathematics writing. In addition, it is important to know how mathematics-writing assessments can be used and scored to gauge the improvement of student knowledge as it relates to mathematics. Finally, it is important to know how teacher and student motivation may play a role in when and how instruction is provided and how students respond to such instruction.

### 1.4 Understanding Motivation and Affect for Mathematics Writing

Student motivation is likely to impact performance on mathematics-writing assessments and the effectiveness of mathematics-writing interventions as an instructional approach. Both writing and mathematics are well known as school-based skill areas in which students are likely to have low self-efficacy and high anxiety that may impact performance (e.g., Brunning & Horn, 2000; Graham & Harris, 2005; Jansen et al., 2013; Pajares & Johnson, 1994; Sanders-Reio, Alexander, Reio, & Newman, 2014; Zimmerman & Bandura, 1994). It is not unreasonable to expect that the aversion to these subjects may be compounded when combining the skill areas by having students write in mathematics. It may be that students with writing anxiety have less anxiety when writing in mathematics if they have strengths in this area. Similarly, it may be that students with mathematical anxiety are less anxious when writing in mathematics if general writing is stronger. Thus, it is important to examine the beliefs and feelings students have about mathematics writing in order to determine how these factors influence mathematics-writing assessments and interventions.

### 1.5 Purpose and Research Questions

Many teachers admit the need for the inclusion of writing in the mathematics classroom but express an inability as to how to combine writing and mathematics (Fukawa-Connelly & Buck, 2010; McCarthy, 2008). As stated, mathematics writing can help students communicate about mathematics and improve knowledge by providing opportunities to explore misconceptions and procedural mistakes; organize thinking and connect mathematical ideas; and develop a deeper and richer understanding of mathematics (Cross, 2009; Thompson, 2010; Verlaan, 2009). Over the past two decades, researchers have conducted numerous studies and analyses about writing (Juzwik et al., 2006), and many authors have written about the importance of mathematics writing (Burns, 2004; Carter, 2009; Ediger, 2006; Fello & Paquette, 2009; Fersten, 2007; Fisher & Frey, 2013; Fukawa-Connelly & Buck, 2010; Lynch-Davis, 2011; Morris, 2006; Parker & Breyfogle, 2011; Sanders, 2009; Staal & Wells, 2011; Thompson, 2010; Wilcox & Monroe, 2011) without providing empirical data on mathematics writing.

The primary goal of this synthesis was to develop an understanding of the collection of empirical studies about mathematics writing for students across the elementary, middle, and high school grades. We wanted to learn how teachers and researchers assess the ability of students to write in mathematics (i.e., assessment), the ways that teachers teach mathematics writing or use mathematics writing to improve students' mathematical skills (i.e., intervention), and the motivation and feelings teachers and students have about mathematics writing (i.e., surveys). After conducting a formal search of the literature, we realized that empirical studies of mathematics writing were sparse and diverse. Therefore, a meta-analysis of the literature was not appropriate. Thus, we included any empirical literature in this synthesis, including qualitative literature that provided some quantitative data. The research questions guiding this synthesis were as follows:

1. What is the empirical research base for mathematics-writing assessments? That is, what forms of mathematics-writing assessments are used, which mathematics topics are assessed using mathematics writing, and how are these assessments scored?
2. What is the empirical research base for mathematics-writing interventions? That is, what types of instructional activities are used, how are the activities implemented, and what is the impact of these activities?

3. What is the empirical research base for the beliefs and feelings teachers and students report about mathematics writing that may impact how they approach these tasks?

## 2. Method

Our research questions provided the basis on which we developed our initial inclusion and exclusion criteria, as well as the foundation for our coding and analysis. At times, studies were collected that tested the limits of our initial plans for our criteria, coding schemes, and analyses; we refined them accordingly. However, all decisions were anchored by our research questions. In this section, we include our inclusion and exclusion criteria, the literature search procedure and PRISMA statement, a table summarizing the studies included in this manuscript, and our coding categories and procedures.

### 2.1 Inclusion and Exclusion Criteria

Studies that included empirical data related to mathematics writing were eligible for inclusion. We selected studies based on the following inclusion criteria:

1. The study required students to write about or in mathematics and:
   a. Analyzed or scored mathematics writing as an assessment;
   b. Analyzed the influence of a mathematics-writing intervention on mathematics or writing outcomes;
   OR
   c. Surveyed students or teachers about their knowledge, motivation, or affect related to mathematics writing.
2. The report was published in English.
3. The report was published in a peer-reviewed journal.
4. The study was conducted with school-age participants in grades 1 through 12, with the exception of surveys, which could include teachers of students in grades 1 through 12.
5. The study was conducted between January 1990 and January 2016.

### 2.2 Literature Search

A multi-step process was used to conduct a comprehensive search of the literature in January of 2016. The first three authors conducted all the searches and coding. Initially, we conducted electronic searches of four databases (i.e., Academic Search Complete, Education Source, Education Resources Information Center (ERIC), and PsycINFO) to identify relevant studies with electronic records between January 1990 and January 2016. We selected 1990 as the start date of the search because this was first year after the release of the NCTM curriculum standards (1989) in which mathematics writing was encouraged as an instructional practice. For the electronic searches, we used the following search terms: math* + writ* + instruct*; math* + writ* + assess*; and math* + writing. This resulted in a total of 6,737 studies; 5,131 after removing duplicates.

Second, we read each title and abstract for evidence of mathematics writing. Overall, 174 studies demonstrated some evidence of mathematics writing, and we obtained the published form of each of these studies. We were able to locate published copies 170 of the 174 studies.

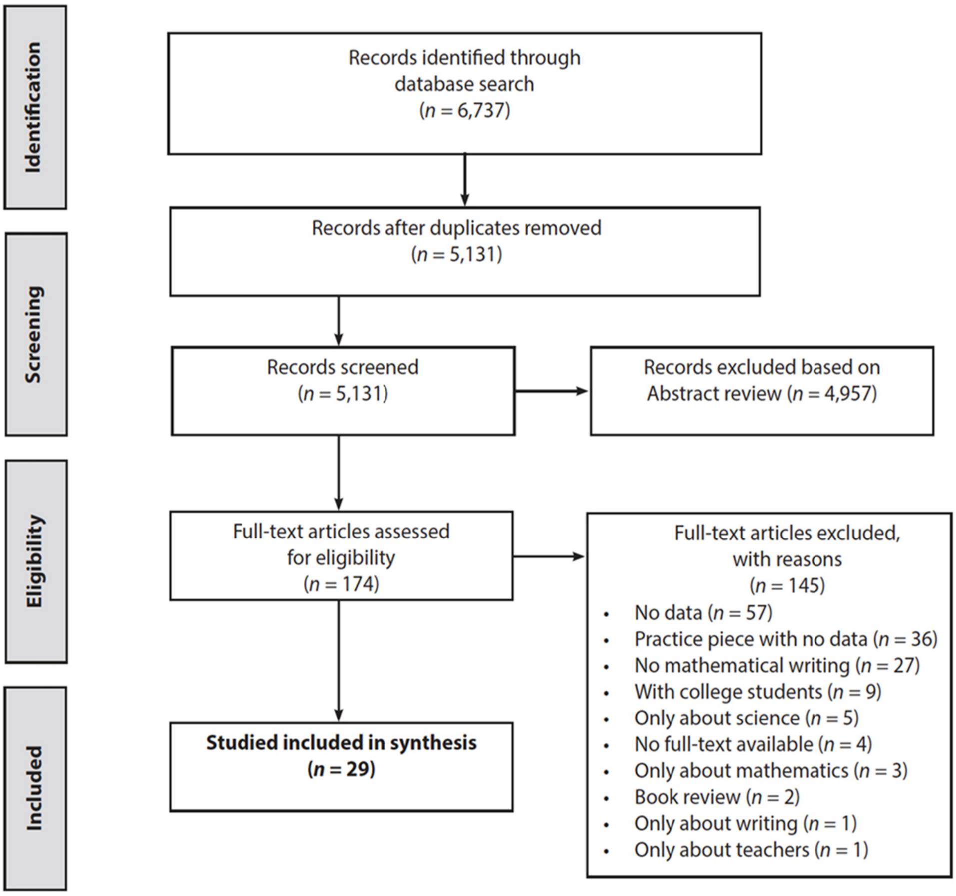


*Figure 1.* PRISMA flow diagram of search process.

Third, we divided the 170 studies among the authors so that two authors reviewed each publication and determined whether the study fulfilled each of the five inclusion criteria. Each author reviewed assigned studies separately and then compared results with another author. When the two authors disagreed about whether a study should be included, all three authors discussed the study to come to a consensus about whether it should be included. Altogether, 29 manuscripts met all inclusion criteria. For more detail about the search process, see the PRISMA flow diagram in Figure 1 (Moher, Liberati, Tetzlaff, & Altman, 2009.) Table 1 displays overall characteristics of the 29 studies.

***Table 1***: Summary of Studies ($N$ = 29)

| Characteristic | *n* | % |
|---|---|---|
| Publication year | | |
| 1990s | 6 | 20.7 |
| 2000s | 12 | 41.4 |
| 2010–present | 11 | 38.9 |
| Study location | | |
| United States | 15 | 51.7 |
| International | 14 | 48.3 |
| Grade level[a] | | |
| Elementary (K–5) | 7 | 24.1 |
| Middle (6–8) | 12 | 41.4 |
| High (9–12) | 14 | 48.3 |
| Participants type[a] | | |
| Full-range of students | 20 | 69.0 |
| High-achieving students | 4 | 13.8 |
| Students at-risk or with disability | 3 | 10.3 |
| Only teachers | 3 | 10.3 |
| Sample size | | |
| <25 | 8 | 27.6 |
| 26 to 50 | 7 | 24.1 |
| 51 to 100 | 4 | 13.8 |
| >100 | 10 | 34.5 |
| Study type | | |
| MW assessment (only) | 8 | 27.6 |
| MW intervention (only) | 4 | 13.8 |
| MW survey (only) | 4 | 13.8 |
| MW assessment + intervention | 9 | 31.0 |
| MW intervention + survey | 3 | 10.3 |
| MW assessment + intervention + survey | 1 | 3.4 |

Note. K = Kindergarten; MW = Math writing.

[a]Several studies included in more than one category.

### 2.3 Coding

After identifying the 29 studies with empirical data about mathematics writing, we categorized each study as containing a mathematics-writing assessment, intervention, or survey. (Note: several studies fell into more than one category). We categorized *assessments* as any mathematics-writing activity that was analyzed or scored. *Interventions* were any classroom activity introduced or led by the teacher in which students engaged in mathematics writing. Surveys included questions asked (of teachers or students) about mathematics writing. We kept each of these definitions quite broad to capture as much empirical data for this synthesis as possible.

Teams of two authors coded each of the 29 studies. For all studies, we coded information in three categories: study descriptors (e.g., grade level of participants), features of the mathematics writing (e.g., mathematical content of the writing), and any empirical data provided. We further coded each study according to features related to the specific category of the study (i.e., assessment, intervention, or survey); all of the specific variables coded can be found in the tables of results for each type of study.
Reliability of coding was calculated by dividing the number of agreements (1139) by the total number of coding decisions (1192). Reliability of coding was 95.6%. To rectify the 53 discrepancies, a third author provided input and consensus was reached through discussion among the three authors.

One of the categories for coding was type of mathematics writing. In October 2015, a task force, funded by the National Science Foundation, of 25 researchers and educators met to identify purposes for and types of mathematics writing and to suggest instructional priorities (Casa et al., 2016). The Elementary Mathematical Writing Task Force identified four types of mathematics writing: (a) exploratory; (b) informative and explanatory; (c) argumentative; and (d) creative. We used this framework to gather information about the types of mathematics writing embedded within current assessments and interventions. Exploratory mathematics writing might be in the form of pre-writing, and it allows students to make sense of problems or ideas. Informative and explanatory writing provides students with the opportunity to describe or explain mathematics. With argumentative mathematics writing, students construct or critique arguments. Finally, with mathematically creative writing, students may write about original ideas that have not been taught or elaborate upon ideas and convey fluency and flexibility in thinking.

## 3. Results

We identified 29 studies for synthesis. Of these, 18 included mathematics-writing assessment data, 17 included a mathematics-writing intervention, and nine collected mathematics-writing opinion or practice information from surveys. We present the mathematics-writing data about assessments, interventions, and surveys in the following three sections.

### 3.1 Assessments

With assessments, we asked what forms of mathematics-writing assessments were used, which mathematics topics were assessed using mathematics writing, and how these assessments were scored. Of the 18 studies with a mathematics-writing assessment, three included elementary students (i.e., grades 1 through 5), 15 included secondary students (i.e., grades 6 through 12), and two included both elementary and secondary students. Half of the studies were conducted in the United States. Only two studies involved students with disabilities: one included students with learning disabilities (Baxter, Woodward, & Olson, 2005) and one contained students with emotional and behavioral disorders (Hauth, Mastropieri, Scruggs, & Regan, 2013). Only five studies reported reliability statistics. Table 2 displays descriptive characteristics of the studies with mathematics-writing assessments.

Nearly half of the studies utilized an assessment to examine the effectiveness of a mathematics-writing intervention. In those cases, the studies were also included in the intervention section of this manuscript. In this section, we focus on the forms, topics, and scoring of mathematics-writing assessments, but we do not to examine differences between student groups. Thus, for studies involving multiple groups, we collapsed the samples to describe how the writing was assessed, as well as the results of the assessment.

**Forms of assessment**. For three-fourths of the mathematics-writing assessments, students participated in explanatory writing; that is, students explained how they solved a problem or how someone could solve a problem. In two studies, students participated in argumentative writing. In one case, students wrote both explanations and arguments in journals at least once a week throughout the school year (Baxter et al., 2005); in the other, students wrote one persuasive essay to a friend to argue the following of order of operations (Hauth et al., 2013). Glogger, Schwonke, Holzäpfel, Nückles, and Renkl (2012) engaged students in pre-writing and explanatory writing by encouraging students to organize ideas (i.e., pre-writing) and elaborate upon the ideas with writing in journals (i.e., explanatory writing). Creative writing was utilized by Barlow and Drake (2008) when students wrote word problems about fractions.

Over half of the studies used journals for collection of student writing. In three instances, students wrote correspondences to real or pseudo students or teachers (Hauth et al., 2013; Norton & Rutledge, 2010; Shield & Galbraith, 1998). We identified no studies in which researchers described having students type the mathematics writing using a typewriter or computer.

**Mathematics topics.** Students wrote about a wide variety of mathematics topics. In the elementary grades, students wrote about operations or geometry and measurement. In the middle school grades, topics expanded to rational numbers, statistics, or algebra.

**Table 2**

*Mathematics-Writing Studies Including an Assessment*

| Study | Location | Grade | Students | *n* | Content | Assess. Type | MW Type | MW Assess. | Score Type | Score Description | Time | Reliability | Results | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Barlow & Drake (2008) | U.S. | 6 | FR | 45 | Fractions | RC | C | Wrote a word problem | Cat. | (a) Omitted answer<br>(b) Incorrect problem representation<br>(c) Minimal (correct math, no word problem)<br>(d) Partial (correct problem, no answer)<br>(e) Satisfactory (correct, unrealistic scenario)<br>(f) Extended (correct, realistic scenario) | SA | NR | (a) 16%<br>(b) 67%<br>(c) 2%<br>(d) 9%<br>(e) 4%<br>(f) 2% | |
| Baxter et al. (2005)[I] | U.S. | 7 | LD; HA | 8 | NR | RC | E; A | Wrote explanations and arguments, along with feelings, in journals | Cat. | (a) Don't know<br>(b) Recording information<br>(c) Summarizing in own words<br>(d) Generalizing to use math ideas<br>(e) Relating connections between concept<br>(f) Affective response<br>(g) Affective dialogue | OT | NR | (a) 11%<br>(b) 31%<br>(c) 31%<br>(d) 10%<br>(e) 0%<br>(f) 11%<br>(g) 6% | |
| Cohen et al. (2015)[I] | U.S. | 2 | FR | 384 | Geometry; Measurement | RC | E | Wrote explanations for answers to math problems | Feat. & Rubric | (a) Reasons (3-point rubric)<br>(b) Use of linking words<br>(c) Formal vocabulary count<br>(d) Informal vocabulary count<br>(e) Formal vocabulary used correctly<br>(f) Informal vocabulary used correctly<br>(g) Attempt at math writing<br>(h) Complete sentences | Post | IRR = .89-.97 | Count:<br>(b) *M* = 2.53 (*SD* = 1.37)<br>(c) *M* = 4.91 (*SD* = 2.35)<br>(d) *M* = 2.37 (*SD* = 1.62)<br>(e) *M* = 3.42 (*SD* = 2.87)<br>(f) *M* = 1.56 (*SD* = 1.51)<br>(g) *M* = 2.89 (*SD* = 1.13)<br>(h) *M* = 3.69 (*SD* = 0.71) | Quality (1-3 points):<br>(a) *M* = 2.20 (*SD* = 1.76) |
| Evens & Houssart (2004) | England | 5 | FR | 441 | Number Sequences | STD | E | Judged correctness and wrote explanations | Cat. | (a) Nothing on script<br>(b) Wrong or irrelevant<br>(c) Restatement<br>(d) Examples given/tested<br>(e) Some degree of justification | SA | NR | (a) 8%<br>(b) 24%<br>(c) 17%<br>(d) 9%<br>(e) 42% | |
| Glogger et al. (2012)[I] | Germany | 9 | FR | 236 | Probability | RC | P; E | Wrote about organization, elaboration, and metacognition in journals | Cat. & Rubric | (a) Rehearsal (copying, example)<br>(b) Organization<br>(c) Elaborations (link new and prior knowledge)<br>(d) Metacognitive | OT | IRR = .76-.90 | Count:<br>(a) *M* = 10.37 (*SD* = 8.17)<br>(b) *M* = 8.91 (*SD* = 7.72)<br>(c) *M* = 7.92 (*SD* = 5.84)<br>(d) *M* = 6.83 (*SD* = 5.71) | Rubric (1-6 points):<br>(a) *M* = 4.39 (*SD* = 1.06)<br>(b) *M* = 3.23 (*SD* = 1.28)<br>(c) *M* = 4.15 (*SD* = 0.92)<br>(d) *M* = 3.14 (*SD* = 1.49) |

**Table 2 (continued)**

*Mathematics-Writing Studies Including an Assessment*

| Study | Location | Grade | Students | *n* | Content | Assess. Type | MW Type | MW Assess. | Score Type | Score Description | Time | Reliability | Results |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hauth et al. (2013) | U.S. | 8 | EBD | 8 | PEMDAS (order of operations) | RC | A | Wrote persuasive essay to friend | Feat. | (a) Total words<br>(b) Total sentences<br>(c) Total paragraphs<br>(d) Total transition words<br>(e) Total essay parts<br>(f) Holistic quality scores | SA | NR | (a) $M = 133.88$ ($SD = 52.35$)<br>(b) $M = 10.63$ ($SD = 4.37$)<br>(c) $M = 2.63$ ($SD = 0.74$)<br>(d) $M = 5.75$ ($SD = 1.04$)<br>(e) $M = 12.12$ ($SD = 1.81$)<br>(f) $M = 9.25$ ($SD = 0.88$) |
| Hoyles & Küchemann (2002) | England | 8, 9 | HA | 266<br>3 | Number theory | STD | E | Judged truth of a math statement and wrote explanations | Cat. | (a) Incorrect/no response<br>(b) No valid justification<br>(c) Correct (incomplete justification)<br>(d) Correct (valid justification) | SA | NR | MW #1:<br>(a) 42%<br>(b) 22%<br>(c) 28%<br>(d) 8%<br>MW #2:<br>(a) 66%<br>(b) 24%<br>(c) 0%<br>(d) 9% |
| Jigyel & Afamasaga-Fuata'i (2007) | Australia | 4, 6, 8 | FR | 55 | Fractions | RC | E | Compared fractions and wrote explanations | Cat. | (a) Correct explanation | SA | NR | (a) By grade level:<br>Grade 4 = 28.6%<br>Grade 6 = 83.3%<br>Grade 8 = 95.5% |
| Jurdak (2008) | Lebanon | 12 | FR or HA | 31 | NR | STD | E | Solved problems and wrote explanations or justifications | Rubric | (a) Math knowledge (0-4 points)<br>(b) Problem solving (0-4 points)<br>(c) Communication (0-4 points) | SA | IRR > .90 | Author created action map that could be used in addition to rubric. |
| Jurdak & Abu Zein (1998)[I;S] | Lebanon | 5, 6, 7[a] | FR | 104 | NR | STD | NR | Wrote answers to math communication items | Rubric | (a) Math communication | Pre-Post | NR | Rubric (0-4 points):<br>(a) Tx = 3.21 ($SD = 0.50$); C = 2.35 ($SD = 0.48$) |
| Kasmer & Kim (2012)[I] | U.S. | MS | FR | 19 | Algebra | RC | E | Responded to prediction questions and wrote explanations | Cat. & Rubric | Prediction responses categorized:<br>(a) Connections to previous knowledge<br>(b) Visualization<br>(c) Connections to previous lessons<br>(d) Use of a math procedure<br>(e) Random guess<br>(f) Indiscernible<br>3-point rubric<br>(f) No understanding (0 points)<br>(g) Partial understanding (1 point)<br>(h) Sophisticated reasoning (2 points) | OT | NR | Categories:<br>(a) 56.2%<br>(b) 60.5%<br>(c) 3.3%<br>(d) 3.9%<br>(e) 1.5%<br>(f) 4.3%<br>Rubric (0-2 points):<br>(g) 17.7%<br>(h) 44.8%<br>(i) 37.5% |

**Table 2 (continued)**

*Mathematics-Writing Studies Including an Assessment*

| Study | Location | Grade | Students | *n* | Content | Assess. Type | MW Type | MW Assess. | Score Type | Score Description | Time | Reliability | Results |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Kostos & Shin (2010)[I] | U.S. | 2 | FR | 16 | Grouping; Addition; Subtraction | STD | E | Predicted patterns and wrote explanations | Rubric | (a) Math knowledge (0-4 points)<br>(b) Strategic knowledge (0-4 points)<br>(c) Explanation (0-4 points) | OT | NR | Rubric (0-12 points):<br>Pretest $M$ = 7.25<br>Posttest $M$ = 10.0 |
| Lim & Pugalee (2004)[I] | Canada | 10 | FR | 12 | Applied Math | RC | E | Wrote in journals to describe and explain | Rubric | (a) Uses clear explanations (level 1-4)<br>(b) Use of math language, vocabulary, and symbols (level 1-4)<br>(c) Selects algorithms and demonstrates computational proficiency using algorithms (level 1-4) | OT | NR | Rubric (3-12 points):<br>Pretest level $M$ = 2.4 ($SD$ = 1.0)<br>Posttest level $M$ = 3.4 ($SD$ = 4.7) |
| Meletiou-Mavrotheris & Paparistodemou (2015) | Cyprus | 6 | FR | 69 | Statistics | RC | E | Wrote answers to questions about applying statistical sampling concepts | Cat. | (a) No response<br>(b) Prestructural (incomplete/idiosyncratic)<br>(c) Unistructural (definitions)<br>(d) Multistructural (sophisticated responses)<br>(e) Relational (integrated) | SA | NR | MW #1: (a) 29% (b) 17% (c) 39% (d) 12% (e) 3%<br>MW #2: (a) 14% (b) 9% (c) 23% (d) 51% (e) 3% |
| Norton & Rutledge (2010)[I] | U.S. | HS | FR | NR | Algebra | RC | NR | Wrote letters about math to pre-service teachers | Cat. | (a) Application<br>(b) Analysis<br>(c) Synthesis<br>(d) Evaluation (comparing methods)<br>(e) Communication<br>(f) Connections (between concepts)<br>(g) Representation (used as cognitive aid)<br>(h) Problem solving (novel situation)<br>(i) Reasoning and proof<br>(j) Problem solving | OT | Kappa Values:<br>(a) 0.67<br>(b) 0.44<br>(c) 0.04<br>(d) -0.01<br>(e) 0.57<br>(f) 0.05<br>(g) 0.39<br>(h) 0.20<br>(i) 0.56 | Study results centered on reliability of the categorizations not student scores. |
| Pugalee (2004)[I] | U.S. | 9 | FR | 20 | Algebra; Problem Solving | RC | E | Wrote descriptions of their problem solving and provided oral descriptions | Cat. | (a) Orientation (to the problem)<br>(b) Organization<br>(c) Execution<br>(d) Verification | OT | IRR = .88 | (a) MW frequency = 71; oral = 38<br>(b) MW frequency = 72; oral = 77<br>(c) MW frequency = 172; oral = 127<br>(d) MW frequency = 21; oral = 44 |

**Table 2 (continued)**

*Mathematics-Writing Studies Including an Assessment*

| Study | Location | Grade | Students | *n* | Content | Assess. Type | MW Type | MW Assess. | Score Type | Score Description | Time | Reliability | Results |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Seo (2009) | U.S. | 10 | FR | 17 | NR | RC | E | Wrote about word problems in math and English classes | Cat. | (a) Definition or directive to accompany equations<br>(b) Incorporation of numbers and symbols as nouns<br>(c) Incorporation of numbers and symbols as verbs | OT | NR | (a) 33%<br>(b) 27%<br>(c) 40% |
| Shield & Galbraith (1998)[I] | New Zealand | 8 | FR | 75 | Integers; Percentages | RC | E | Wrote explanations to a student absent from class and a student having difficulty | Cat. | (a) Linked new procedure to prior knowledge<br>(b) Talked through a procedure algorithmically<br>(c) Goal, kernel, worked example | OT | NR | (a) 4%<br>(b) 33%<br>(c) 35% |

*Note.* A = Argumentative mathematical writing; Assess. = Assessment; C = Creative mathematical writing; Cat. = Category; E = Explanatory mathematical writing; EBD = Emotional/behavioral disorder; Feat. = Feature; FR = Full range; HA = High achieving (including gifted); HS = High school; IRR = Inter-rater reliability; LD = Learning disability; MS = Middle school; MW = Mathematical writing; NR = Not reported; OT = Over time; P = Pre-writing in mathematics; Post = Posttest; Pre-Post = Pretest and posttest; RC = Researcher created; SA = Stand alone; STD = Standardized; U.S. = United States.

[a]Calculated based on ages reported by authors.

[I]Study also included intervention data (see Table 3).

[S]Study also included survey data (see Table 4).

In high school, students wrote about algebra and problem solving. In several studies, the mathematics content was not divulged by the authors. We assume content included grade-specific mathematics concepts and procedures, especially when assessment data were gathered over a number of weeks and months.

**Scoring of assessments.** After examining mathematics-writing assessments, we organized the scoring of assessments into three subgroups: categories, features, or rubrics.

**Response categories.** Two-thirds of assessment studies (*n* = 12) attempted to qualitatively classify student mathematics writing according to categories. The categories used by researchers varied, but all of the categorizations centered around identifying the cognitive level of understanding demonstrated by the student in any particular statement within the mathematics writing. In many cases, mathematics writing was scored as fitting into one category. For example, Hoyles and Küchemann (2002) scored mathematics writing as falling into one of four categories: incorrect, no valid justification, correct with incomplete justification, or correct with valid justification. Only 8% of students provided a correct explanation with valid justification. In other cases, mathematics writing was scored according to the frequency of occurrence within categories. For example, Pugalee (2004) categorized individual statements as: orientation to the problem, organization, execution, and verification. Across studies, categorical scoring demonstrated that students often engaged in mathematics writing that involved copying or restatement (Glogger et al., 2012) rather than elaborating upon new ideas (Shield & Galbraith, 1998). Students provided incorrect or incomplete justifications more often than correct and elaborated explanations (e.g., Evens & Houssart, 2004; Meletiou-Mavrotheris & Paparistodemou, 2015). In the only work comparing performance across grade levels, Jigyel and Afamasaga-Fuata'i (2007) scored mathematics writing at fourth, sixth, and eighth grade according to whether the students wrote a correct explanation. As grade level increased, correct explanations increased from less than 30% to above 95%.

**Features.** Two of the 18 studies examined features of the students' mathematics writing (Cohen, Miller, Casa, & Firmender, 2015; Hauth et al., 2013). The types of features included both traditional counts of writing (e.g., number of words, number of sentences, transition words) and aspects of writing related to mathematical language (e.g., informal and formal mathematical vocabulary). Scores indicated that some traditional writing features were used when students wrote in mathematics. Students used sentences, paragraphs, transition or linking words, and introductions in mathematics writing (instead of simply writing notes or shorthand in mathematics). For example, Hauth et al. (2013) determined that eighth-grade students, when asked to write a persuasive essay about order of operations, wrote approximately 134 words using almost 11 sentences and nearly three paragraphs. Students used approximately six transition words per mathematics-writing sample, which was more than twice the number of transition words used by second-grade students in Cohen et al. (2015). It is likely the grade level of the students influenced this difference. Interestingly, Cohen et al. determined that the average mathematics-writing explanation included almost five mathematical vocabulary terms, yet only one or two of these terms were used correctly.

**Rubrics.** Seven of the 18 studies attempted to score mathematics-writing quality using a rubric. Two studies used a 3-point rubric (Cohen et al., 2015; Kasmer & Kim, 2012), one used a 4-point rubric (Lim & Pugalee, 2004), three used 5-point rubrics (Jurdak, 2008; Jurdak & Abu Zein, 1998; Kostos & Shin, 2010), and one used a 6-point rubric (Glogger et al., 2012). Two studies utilized rubrics in combination with categorical scoring; one study combined rubric scoring with features scoring. Both Kostos and Shin (2010) and Lim and Pugalee (2004) used rubrics to analyze student mathematics writing at pre- and posttest (i.e., after implementation of a mathematics-writing intervention). Scoring from the rubrics indicated an increase in scores from pre- to posttest. Jurdak and Abu Zein (1998) used a rubric to analyze mathematics-writing differences between experimental and control conditions, and rubric scores indicated performance differences favoring students in the experimental condition.

### 3.2 Interventions

We explored the empirical research base for mathematics-writing interventions by asking what types of instructional activities were used, how the activities were implemented, and whether the activities had an impact on mathematics, writing, or mathematics writing. Of the 29 studies in this synthesis, 17 included a mathematics-writing intervention. We defined intervention as an instructional activity implemented by the classroom teacher or researcher in which students wrote about or in mathematics. In some cases, the mathematics writing created by students during the intervention was scored. If so, we described the scoring methods in the previous section related to assessment (see Table 2). In other cases, the mathematics writing was not scored or researchers employed alternative methods to evaluate the effectiveness of the intervention (e.g., survey, mathematical assessment). Table 3 provides a summary of the characteristics of each intervention.

**Table 3**
*Mathematics-Writing Studies Including an Intervention*

| Study | Location | Grade | Students | Participants by Group | | | | Math Content | Study Type | MW Type | MW Intervention | Implement | Assessment | Results |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | *$n_1$* | *$n_2$* | *$n_3$* | *$n_4$* | | | | | | | |
| Baxter et al. (2005)[A] | U.S. | 7 | LD; HA | 4 | 4 | | | NR | Case | E; A | $Tx_1$: Journal writing at least 1/week for 30 weeks<br>C: Journal writing at least 1/week for 30 weeks; comparison group with HA | Teacher | MW | NR; $Tx_1$ had higher frequency of "I don't know" answers and of recording and summarizing; C used MW to develop solutions |
| Cohen et al. (2015)[A] | U.S. | 2 | FR | 193 | 191 | | | Measurement; Geometry | QExp | P; E; A | $Tx_1$: Two 6-week units which required MW<br>C: No MW | Teacher | MW | $Tx_1$ > C in ability to provide reasoning and correctly use math vocabulary |
| Cross (2009) | U.S. | 9 | FR | 43 | 51 | 62 | 55 | Algebra | QExp | E; A | $Tx_1$: 10 weeks of argumentative intervention (read question, explain response aloud, defend)<br>$Tx_2$: 10 weeks of MW to explain why and how solution was correct<br>$Tx_3$: 10 weeks of argumentative + MW<br>C: No MW or argumentative | Teacher | Math test | $Tx_3 = Tx_2$<br>$Tx_3 > Tx_1$<br>$Tx_3$ > C |
| Fortescue (1994)[S] | U.S. | 3 | FR | NR | | | | Multiplication | Qual | E | $Tx_1$: Journal writing about math menu activities provided by teacher | Teacher | Survey | NR; 70% of $Tx_1$ students felt MW helped them learn |
| Glogger et al. (2012)[A] | Germany | 9 | FR | 236 | | | | Probability | Case | E | $Tx_1$: Journal writing for 6 weeks | Teacher | MW | $Tx_1$ demonstrated aspects of MW |
| Idris (2009)[S] | Malaysia | 10, 11[a] | FR | 85 | 86 | | | Calculus | QExp | E | $Tx_1$: MW 5/week for 5 weeks<br>C: No MW | Teacher | Math test | $Tx_1$ > C |
| Jurdak & Abu Zein (1998)[A,S] | Lebanon | 5, 6, 7[a] | FR | 52 | 52 | | | NR | Exp | E | $Tx_1$: Journal writing for 7-10 min, 3/week for 12 weeks<br>C: No MW | Teacher and Researcher | Math test | $Tx_1$ > C |
| Kasmer & Kim (2012)[A] | U.S. | MS | FR | 19 | | | | Algebra | Case | O | $Tx_1$: Wrote responses to prediction questions | Teacher and Researcher | MW | $Tx_1$ judged against a rubric: 18% scored 0, 45% scored 1, 37% scored 2 |
| Kostos & Shin (2010)[A] | U.S. | 2 | FR | 16 | | | | Addition; subtraction | Qual | E | $Tx_1$: 16 journal writing experiences over 5 weeks, teachers provided modeling and mini-lessons for incorporating math vocabulary | Researcher as teacher | MW | $Tx_1$ demonstrated significant growth from pretest |
| Liedtke & Sales (2001)[S] | Canada | 7 | FR | 31 | | | | NR | Case | E; CR | $Tx_1$: MW encouraged in class over an entire school year | Teacher | Survey | $Tx_1$ affect improved from the beginning of the year |

**Table 3 (continued)**
*Mathematics-Writing Studies Including an Intervention*

| Study | Location | Grade | Students | Participants by Group $n_1$ | $n_2$ | $n_3$ | $n_4$ | Math Content | Study Type | MW Type | MW Intervention | Implement | Assessment | Results |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Lim & Pugalee (2004)[A] | Canada | 10 | FR | 12 | | | | Problem solving | Qual | E | $Tx_1$: Journal writing for 10 min at end of class a few times a week for the semester | Teacher | MW | $Tx_1$ scores improved from the beginning of the year |
| Miller (1993) | U.S. | 9 | FR | 50 | | | | Algebra | Case | E; CR | $Tx_1$: MW about 55 prompts given over the semester; 5 min to read prompt and respond | Teacher | MW | NR; detailed discussions of 3 prompts |
| Moran et al. (2014) | U.S. | 3 | AR | 16 | 19 | 24 | 13 | Problem solving | Exp | O | $Tx_1$: 20 lessons of restate MW; 10 weeks, 25-30 min<br>$Tx_2$: 20 lessons of relevant MW; 10 weeks, 25-30 min<br>$Tx_3$: 20 lessons of complete MW; 10 weeks, 25-30 min<br>C: No MW | Tutor | Math test | $Tx_3 = Tx_2 > Tx_1 = C$ |
| Norton & Rutledge (2010)[A] | U.S. | HS | FR | NR | | | | Algebra | Qual | O | $Tx_1$: Wrote letters back and forth to pre-service teachers for 5 weeks or 7 weeks | Teacher | MW | $Tx_1$ scores increased and then slightly decreased |
| Pugalee (2004)[A] | U.S. | 9 | FR | 20 | | | | Algebra | Counter | E; A | $Tx_1$: Journal writing for 2 weeks | Teacher | MW | NR; detailed discussion about scoring |
| Shield & Galbraith (1998)[A] | New Zealand | 8 | FR | 75 | | | | Integers; Percentages | Qual | E | $Tx_1$: Letter writing for 3 months to a friend who was absent from class or a student with math difficulty | Researcher | MW | NR; coding scheme developed |
| Tan & Garces-Bascal (2013)[S] | Singapore | 7 | HA | 27 | 27 | | | Algebra | QExp | E | $Tx_1$: Journal writing for 5-10 min 1/week for 6 weeks<br>C: No MW | Researcher as teacher | Math test | $Tx_1 > C$ |

*Note.* A = Argumentative mathematical writing; AR = At-risk for learning disability; Case = Case Study; Counter = Counterbalanced design; C = Creative math writing; E = Explanatory mathematical writing; Exp = Experiment; FR = full range; HA = High achievers (including gifted); HS = High School; LD = Learning Disability; MS = Middle School; MW = Mathematical writing; NR = Not Reported; O = Other; P = Pre-writing in mathematics; QExp = Quasi-experiment; Qual = Qualitative; U.S. = United States.
[a]Calculated based on ages reported by authors.
[A]Study also included assessment data (see Table 2).
[S]Study also included survey data (see Table 4).

**Types of intervention activities.** Similar to the collection of studies with a mathematics-writing assessment, over three-fourths of the intervention studies focused on explanatory mathematics writing. In four studies, students also engaged in argumentative mathematics writing. In two studies, students worked on explanatory and creative writing. Three of the interventions asked students to write in mathematics using forms not outlined by Casa et al. (2016). That is, students wrote responses to predictions about algebraic equations (Kasmer & Kim, 2012), learned to rewrite word problems (Moran, Swanson, Gerber, & Fung, 2014), or engaged in pen-pal correspondences about mathematics to pre-service teachers (Norton & Rutledge, 2010). Across all of these purposes, writing occurred in one of three contexts: journals, correspondences, or organized mathematics writing in the classroom. Journal writing was the most popular context, with approximately half of the interventions employing journal writing.

**Implementation of intervention activities.** With journal writing, the minimum length of intervention was 2 weeks, and the maximum length was 30 weeks. Most journal writing occurred with some frequency. That is, students wrote in mathematics journals at least once a week (e.g., Tan & Garces-Bascal, 2013) or several times a week (e.g., Jurdak & Abu Zein, 1998). In the two studies with correspondence writing to others, students wrote to a pre-service teacher (Norton & Rutledge, 2010) or to a pseudo student (Shield & Galbraith, 1998). Students participated in the correspondence writing for 5 weeks to 3 months. In the remaining seven studies, teachers engaged students in mathematics-writing interventions with writing occurring within mathematics coursework. This writing was neither in a journal or correspondence format but organized mathematics-writing practice opportunities provided by the classroom teacher. These activities lasted from 5 weeks to the entire school year.

**Impact of intervention activities.** We report the impacts of instructional activities across the contexts used for mathematics writing.

- ***Journals***. Five studies provided empirical evidence supporting the use of journal writing in mathematics. For example, in two studies (Jurdak & Abu Zein, 1998; Tan & Garces-Bacsal, 2013), a control group was used to determine the efficacy of writing in mathematics journals. In both studies, the outcome measure was a mathematical assessment, not a mathematics-writing measure, and students who participated in journal writing outperformed students who did not. A third study (Baxter et al., 2005) compared the use of mathematics journals with four students with learning disabilities to a control group of higher-performing students without disabilities; the control group's purpose was not to determine the efficacy of the intervention. Two studies with journal writing (Kostos & Shin, 2010; Lim & Pugalee, 2004) determined that student mathematics-writing scores improved from pre- to posttest, yet researchers did not employ a control group for direct comparisons about the efficacy of the intervention. Results from the final three journal studies were less empirical. Glogger et al. (2012) merely reported that students demonstrated using aspects of mathematics writing after writing in mathematics journals; Fortescue (1994) stated that a survey of students who used journals for mathematics writing showed they believed the journaling helped with learning and Pugalee (2004) did not report results but provided a comparison of writing in mathematics journals to thinking aloud.

- *Correspondences.* Students corresponded in mathematics in two studies. Norton and Rutledge (2010) reported initial gains in mathematics-writing scores. Across the weeks, mathematics-writing scores of students initially increased slightly and then decreased. The authors attributed this pattern to the novelty of mathematics correspondence writing wearing off. Shield and Galbraith (1998) did not report any empirical results about the effectiveness of the intervention.

- *Organized classroom writing.* In the remaining seven studies, teachers engaged students in mathematics-writing interventions with writing occurring within mathematics coursework. Four of these interventions used treatment and control groups, and results favoring the mathematics-writing interventions were positive and significant. For example, Cohen et al. (2015) asked second-grade students to write about geometry and measurement within a 12-week intervention. At posttest, students who wrote in mathematics had significantly higher scores on mathematical reasoning and use of mathematical vocabulary than students in the control group. At the high school level, Idris (2009) asked students in a calculus course to explain or describe how to solve different mathematical problems on a daily basis for 5 weeks. At posttest, students who wrote in mathematics outperformed students in the control group on an assessment of mathematics. Cross et al. (2009) also determined performance differences on a mathematical assessment with high school students. Students in a 10-week intervention with a combination of learning how to develop an oral argument and write an explanation outperformed students in a control group and those who received intervention about the oral argumentation only. The other study with a control group focused on at-risk learners (Moran et al., 2014). Students in three active interventions learned to write in mathematics by paraphrasing or by using key terms and information. On a mathematical assessment, students who focused on key terms and information outperformed those who learned to paraphrase and those in a control group.

  Three studies did not employ control groups to understand the impact of a mathematics-writing intervention. Miller (1993) and Liedtke and Sales (2001) encouraged students to write in mathematics for a semester or the entire school year. Student affect about mathematics improved, but no other information was provided about student improvement as it related specifically to mathematics writing. Kasmer and Kim (2012) asked students to construct responses to prediction questions posed in a middle school classroom. Again, the authors did not report growth scores from pre- to posttest.

### 3.3 Surveys

For surveys, we asked about the beliefs and feelings that teachers and students reported about mathematics writing. Of the 29 studies, nine administered a mathematics-writing survey with three teacher surveys and six student surveys. Table 4 displays details about the sample for each survey, as well as the survey questions and results.

**Teacher surveys.** We identified three surveys of teachers that included questions about mathematics-writing practices. One involved understanding how writing practices in mathematics differed from those in other content areas in middle and high school (Applebee & Langer, 2011). Teachers reported assigning writing less often in mathematics courses compared to English, science, and history courses. Teachers also reported spending little time providing instruction about writing in mathematics compared to the other subject areas. The second survey asked high school teachers, including mathematics teachers, how often they used specific writing practices to support student learning in the classroom (Gillespie, Graham, Kiuhara, & Hebert, 2013). In terms of writing, mathematics teachers assigned mathematics writing less often than all other categories of high school teachers (i.e., language arts, science, and social studies) in the survey. The mathematics teachers, however, used writing to solve a problem and step-by-step instructions significantly more often than other teachers. The writing tasks mathematics teachers reported using (at least several times a year) by the highest percentages were explanations (93%), writing to solve a problem (91%), and step-by-step instructions (85%). Interestingly, although a large percentage of the intervention studies identified for the current synthesis involved journal writing, 73% of mathematics teachers in the Gillespie et al. (2013) survey reported never using journal writing in their classrooms. Swinson's (1992) survey asked high school teachers how often writing prompts, journaling, letter writing (i.e., correspondences), summarizing, essay writing, and rewriting were used in mathematics classes. The majority of teachers reported no use of journal writing (86%) or correspondences (95%). Approximately half of teachers never used writing prompts to encourage mathematics writing, and 62% of teachers never utilized essay writing. The writing practices with some regular use included summarizing and rewriting. Of the high school teachers, 18% reported use of summarizing at least 5 to 10 times a term, and 10% used summarizing each week. Similarly, 14% of teachers used rewriting 5 to 10 times a term, and 10% used rewriting each week.

**Student surveys.** One student survey was completed with elementary students, and five were conducted with middle and high school students. In the elementary student survey, Fortescue (1994) asked students in a third-grade classroom if writing in

**Table 4**

*Mathematics-Writing Studies Including a Survey*

| Study | Location | Grade | *n* | MW survey questions[a] | Results |
|---|---|---|---|---|---|
| Applebee & Langer (2011) | U.S. | MS teachers; HS teachers | 1,520[b] | (a) How many writing assignments in a 9-week period?<br>(b) How often do students write at least one paragraph?<br>(c) What are the approaches to writing instruction? | (a) In math, about 1; in English, 9<br>(b) In math, no paragraphs reported in MS but 3.4% of the time in HS; in English, 9-12% of the time<br>(c) In math, compared to English, Science, and History, teachers infrequently spend class time on writing ideas, teacher specific strategies for writing, providing models of writing, providing writing rubrics, and asking students to work together to plan, edit, and revise work. |
| Clarke et al. (1993) | Australia | 7, 8, 9, 10, 11, 12 | 150 | Questions with respect to:<br>(a) journal use<br>(b) journal purpose<br>(c) journal difficulties<br>(d) journal value<br>(e) teacher actions | (a) 54% reported MW in journals after every lesson; 90% reported reading their journals<br>(b) NR<br>(c) Even division over whether it was difficult to put math thinking into words<br>(d) 60% used journals "because it helps me" or "to help me learn"<br>(e) Students estimated teachers read journals once a month or less frequently |
| Fortescue (1994)[I] | U.S. | 3 | NR | (a) Does writing about math help you understand better? | (a) 70% agreed |
| Gillespie et al. (2013) | U.S. | HS teachers | 54[d] | *How often do you use the following writing activities to support learning in your classroom?*<br>(a) *descriptions*<br>(b) *journal entries*<br>(c) *explanations*<br>(d) *summaries*<br>(e) *step-by-step instructions*<br>(f) *writing to solve a problem*<br>(g) *writing to make personal connections*<br>(h) *generating written questions* | never / < weekly / ≥ weekly<br>(a) 17% / 61% / 23%<br>(b) 73% / 20% / 8%<br>(c)[e] 7% / 49% / 44%<br>(d) 40% / 47% / 13%<br>(e) 15% / 51% / 34%<br>(f) 8% / 39% / 52%<br>(g) 70% / 30% / 0%<br>(h) 59% / 41% / 0% |
| Idris (2009)[I] | Malaysia | 10, 11[c] | 85 | (a) *I like calculus better now.*<br>(b) *I learned*<br>(c) *I spent more time on calculus now than before*<br>(d) *I enjoy calculus better now than before*<br>(e) *It was easy to learn calculus by writing activities*<br>(f) *I learn calculus better by reflecting instead of only with book and memorizing* | (a) 79%<br>(b) 84%<br>(c) 85%<br>(d) 87%<br>(e) 83%<br>(f) 83% |
| Jurdak & Abu Zein (1998)[A;I] | Lebanon | 5, 6, 7[c] | 52 | (a) *How has writing effected your learning of mathematics?*<br>(b) *How do you feel about journal writing for this class?*<br>(c) *What are the benefits of journal writing in math classes?*<br>(d) *How could journal writing be changed to be more effective?*<br>(e) *Would you like to continue this activity in math?* | (a) Majority reported MW helped check understanding, acquire knowledge, and study; 11% of students reported no effect<br>(b) All students reported positive feelings about MW<br>(c) Majority listed benefits such as fosters learning, expresses feelings, helps in remembering and reviewing, and improves grades<br>(d) 48% reported no changes; 19% wanted more variety in prompts<br>(e) 92% said they would continue |

**Table 4 (continued)**

*Mathematics-Writing Studies Including a Survey*

| Study | Location | Grade | *n* | MW survey questions[a] | Results |
|---|---|---|---|---|---|
| Liedtke & Sales (2001)[I] | Canada | 7 | 31 | (a) *Writing can be an important part of learning mathematics.* T/F<br>(b) *Sharing ideas in mathematics can involve writing.* T/F<br>(c) *I enjoy writing about mathematics.* T/F<br>(d) *Reading the writing of others can show me different ways of thinking about mathematics.* T/F | Change from pre- to posttest:<br>(a) 16 T to 25 T<br>(b) 10 T to 26 T<br>(c) 1 T to 9 T<br>(d) 13 T to 24 T |
| Swinson (1992) | Australia | HS teachers | 226 | Use of writing activities in mathematics classes:<br>(a) *writing prompts*<br>(b) *journal*<br>(c) *letter writing*<br>(d) *summarising*<br>(e) *essay*<br>(f) *rewriting*<br>(g) *any other* | never / <5 term / 5-10 term / each week<br>(a) 49% / 33% / 11% / 7%<br>(b) 86% / 8% / 4% / 1%<br>(c) 95% / 4% / 1% / 0%<br>(d) 37% / 35% / 18% / 10%<br>(e) 62% / 37% / 1% / 0%<br>(f) 37% / 40% / 14% / 10%<br>(g) 61% / 16% / 9% / 14% |
| Tan & Garces-Bacsal (2013)[I] | Singapore | 7 | 27 | (a) *The journal writing is useful in improving my understanding of the algebra concepts.*<br>(b) *The journal writing has allowed me to reflect on my mistakes effectively.*<br>(c) *The journal writing has improved my rapport with my teacher.*<br>(d) *The feedback given to me through journal writing has been useful for my learning.*<br>(e) *Journal writing has helped my monitor my own thinking such as the selection and use of problem-solving strategies.*<br>(f) *I would like to continue journal writing for other topics in mathematics.*<br>(g) *I would like to recommend journal writing in the teaching of algebra.*<br>(h) *I would like to recommend journal writing for the teaching of other topics in mathematics.* | (a) 93%<br>(b) 89%<br>(c) 79%<br>(d) 89%<br>(e) 82%<br>(f) 72%<br>(g) 86%<br>(h) 75% |

*Note*. HS = High school; MS = Middle school; MW = Mathematics writing; NR = Not reported; T/F = True/false; U.S. = United States.

[a]Italicized text was typed verbatim from published studies.
[b]Not every question answered by the full sample.
[c]Calculated based on ages reported by authors.
[d]Represents the mathematics teachers in the study.
[e]This question was answered by all 210 teachers in the study; only 54 taught mathematics.

[A]Study also included assessment data (see Table 2).

[I]Study also included intervention data (see Table 3).

mathematics helped them understand mathematics better. The majority of students responded in the affirmative. In five studies that surveyed students in middle or high school, all students answered survey questions after participating in a mathematics-writing intervention (e.g., journal writing). In terms of use, some students agreed it was difficult to use words to describe mathematics (Clarke, Waywood, & Stephens, 1993). Students did, however, agree writing was an important part of learning mathematics (Liedtke & Sales, 2001) and would recommend mathematics writing for all mathematics domain areas (Tan & Garces-Bacsal, 2013). For questions about affect and motivation, students reported liking their subject matter more after mathematics writing (Idris, 2009; Jurdak & Abu Zein, 1998; Tan & Garces-Bacsal, 2013), enjoying mathematics writing (Liedtke & Sales, 2001), and wanting to continue using mathematics writing (Jurdak & Abu Zein, 1998; Tan & Garces-Bacsal, 2013). With regard to self-efficacy, students reported that mathematics writing helped them learn (Clarke et al., 1993; Idris, 2009) and helped them monitor their thinking and learning (Tan & Garces-Bacsal, 2013).

## 4. Discussion

A challenge of this synthesis was that researchers were sometimes unclear about the lens through which mathematics writing was examined. That is, researchers did not always describe whether they were attempting to use mathematics writing to assess or improve *mathematics skill* or *writing skill* or whether they were examining *mathematics writing* as its own construct. This is noteworthy, as Bossé and Faulconer (2008) discussed the importance of distinguishing between *writing about* mathematics (e.g., biographies about mathematicians) and *writing in* mathematics (e.g., writing explanations of mathematical concepts). Other researchers have suggested mathematics teachers may view a focus on literacy skills in the mathematics classroom as neglecting, de-emphasizing, or misrepresenting the mathematics or mathematics information (Siebert & Draper, 2008). Our view is that it is possible to use mathematics writing in ways that effectively emphasize (a) mathematics content or skills, (b) writing content or skills, and (c) mathematics writing as its own construct. Researchers and teachers, however, need to be clear about the purpose for using mathematics writing to ensure the research can be effectively translated into practice.

The amount of empirical data across the three types of studies (i.e., assessment, intervention, and survey) was sparse. Therefore, we included qualitative research, action research, and studies published in practitioner journals in order to provide the most comprehensive view of the peer-reviewed empirical literature. Due to this, we were not able to synthesize the data using statistical methods, such as meta-analysis, because the studies varied widely in the methodology, research questions, measures, and analyses. Moreover, the quality of many studies was relatively poor. That is, many assessment studies did not include reports of basic reliability, validity, or inter-observer agreement. Similarly, intervention studies included complications, such as a lack of reported fidelity, lack of random assignment, lack of control group, limited descriptions of the treatment and control conditions, and use of a single teacher per condition, among others. Despite these limitations, there were important trends across this literature that could be used as a foundation for potential future work.

### 4.1 Assessments

On most assessments, students wrote explanations. Far less often did students engage in argumentative or creative mathematics writing. Interestingly, students wrote about a wide variety of mathematics content, which indicates that mathematics writing can be used across grade levels and mathematical domains. In terms of scoring mathematics-writing assessments, researchers used categories, features, rubrics, or a combination of these three approaches. Because the assessment prompts differed based on mathematical topics (e.g., addition, measurement, algebra) and the scoring of prompts varied widely, it is difficult to produce specific conclusions about mathematics-writing assessments. We can, however, provide several discussion points.

Across assessment scoring methods, findings consistently indicated students have room for mathematics-writing improvement. Researchers learned that few students engaged in high-level reasoning, and many students tended to use informal mathematics terminology and had difficulty incorporating mathematical symbols into writing. In multiple studies, students rarely reached ceiling levels on quality measures. Only a handful of the 18 studies collected information about reliability of scoring. All reported reliability was within acceptable ranges, indicating promise for these different methods of scoring. Future research, however, must report reliability measures and explore how educators can become reliable scorers of mathematics writing.

### 4.2 Interventions

Similar to the assessments, the majority of interventions asked students to provide written explanations. Students wrote in journals, wrote correspondences, or engaged in other organized classroom writing activities. A few studies lasted only a few sessions, whereas other studies lasted a semester or entire school year. To measure the effectiveness of the interventions, these studies used mathematics-writing assessments from all three scoring domains (i.e., categories, features, rubrics), as well as measures of mathematics skill that did not involve writing. The intervention studies represented a range of mathematical content, study designs, grade levels, and mathematics-writing types, which makes it difficult to draw conclusions. The majority of intervention studies also did not supply adequate statistics, making it challenging to compute effects for many outcome measures. Despite this, common characteristics of the mathematics-writing interventions emerged in studies that exhibited a positive result.

For interventions involving journal writing, results indicated students benefitted from this practice. In most journal-writing interventions, students wrote in mathematics across multiple sessions across several weeks. Importantly, results from studies with control groups showed improvement on mathematics assessments, indicating writing in mathematics helped increase student knowledge about mathematics content. Several studies also showed that student scores improved on mathematics-writing assessments, demonstrating mathematics writing can improve with practice. The two correspondence-writing interventions did not show consistency with results, and there were too few studies from which to draw conclusions. For organized mathematics writing in the classroom, intervention results were promising. On measures of mathematics writing and mathematics assessments, students who participated in consistently implemented mathematics-writing interventions demonstrated both significant gains over control groups or positive growth from pre- to posttest.

### 4.3 Surveys

Surveys revealed teachers rarely provided students opportunities to write in mathematics compared to other content areas. When teachers did have students write in mathematics, students did not receive instruction on specific mathematics-writing skills such planning, editing, and revising. If, as the Common Core suggests, we expect students to communicate about mathematics and express mathematical ideas using writing, it is necessary for schools to increase the mathematics-writing opportunities for students.

Though teachers reported providing limited opportunities, students conveyed favorable impressions about mathematics writing. Students expressed that use of journals aided their learning of mathematics content and reported that receiving feedback on their mathematics writing was helpful. Across surveys, students stated that they enjoyed writing in mathematics settings and would like to continue mathematics-writing activities. As many students experience anxiety about mathematics or report not liking mathematics, it is important for educators to use activities that engage students; mathematic writing may be one such activity.

### 4.4 Recommendations for Future Research

Given the limited high-quality research across each of the areas of mathematics writing, we provide six recommendations for future research. For each recommendation, we denote whether the recommendation was related to assessments, interventions, or survey research.

**Develop mathematics-writing assessments with consistent, balanced, and integrated scoring.** In this synthesis, researchers and teachers scored mathematics-writing assessments in three ways: *categorization of written statements*, *writing features*, and *writing quality* (using rubrics). One limitation of the current literature is that researchers rarely integrated these methods for scoring and analyzing mathematics writing. Most information gathered from categorizing statements was likely related to the prompt, rendering it difficult to develop conclusions about whether mathematics writing (a) provided an accurate representation of students' mathematical knowledge, (b) was effective at improving mathematical or writing knowledge, or (c) represented student writing ability, mathematical ability, or both. Similarly, examining writing features, such as number of words and number of sentences, provided information about the amount of mathematics-writing content but not the quality of the content. On the other hand, rubrics provided a practical and useful quantitative measure of writing quality, but these scores did not provide an approach to measure the individual influences of mathematical and writing skill. By analyzing mathematics writing with a combination measures (i.e., rubrics and features), researchers could capitalize on multiple sources of information.

Three studies in this synthesis provided strong examples of the utility of this approach. Cohen and colleagues (2015), Glogger and colleagues (2012), and Kasmer and Kim (2012) all examined features of writing, while also incorporating a rubric to measure quality. This combination of scoring methods allowed the researchers (and readers) to examine how mathematics-writing features may have impacted judgments of writing quality.

**Improve the psychometrics of mathematics-writing measures.** In addition to improving construct validity through the first recommendation, efforts also need to be made to report and improve the psychometrics of mathematics-writing measures. Across the 18 assessment studies, 13 did not report reliability, four included information about interrater agreement, and only one reported a measure of internal consistency (i.e., Kappa values; Norton & Rutledge, 2010). Of the studies that reported interrater agreement, all reported reliability within acceptable ranges, indicating promise for scoring consistency. However, Norton and Rutledge (2010), reported Kappa values ranging from -0.01 to 0.67 on their categorizations of student statements, indicating the reliability of scoring varied widely, and many of the scoring categories did not meet internal consistency standards. Future research must include measures of reliability, and measures should meet acceptable reliability standards before such measures are used to examine the effectiveness of interventions or to make instructional decisions. High reliability of scores is also necessary to help to improve the construct validity of assessments. In other words, we need to know whether a mathematics-writing assessment is measuring what it purports to measure.

**Correlate measures with mathematical skills and writing skills, and examine the ability of measures to predict later achievement.** There are essentially two purposes that researchers and teachers report for having students write about or in mathematics. One, communicating in mathematics using writing allows teachers to assess student mathematical understanding. Two, writing in mathematics may improve students' understanding of mathematics, as well as their writing skills. Currently, the literature in this synthesis does not allow us to make strong conclusions about the validity of these assumptions because no studies correlated mathematics writing with mathematical knowledge or general writing skills.

In order to improve the criterion validity of mathematics-writing measures, researchers need to make a concerted effort to correlate mathematics-writing assessments with other measures of mathematical knowledge and general writing skill. Further, it is important to examine whether mathematics-writing performance predicts later mathematical and writing achievement. This may assist researchers and teachers in determining how to apply mathematics-writing assessments. For example, if mathematics writing is correlated with a mathematics measure but does not lead to better mathematical outcomes, teachers may want to simply use mathematics-writing assessments to assess mathematical understanding, instead of attempting to improve mathematics-writing skills. On the other hand, if mathematics-writing performance predicts later mathematical performance, teachers would want to put more effort into improving students' mathematics-writing skills.

**Develop systematic and explicit interventions for mathematics writing.** Although the mathematics-writing interventions demonstrated some promise for improving students' mathematics writing or attitudes about mathematics writing, 13 of the 17 intervention studies (76%) did not report providing systematic or explicit instruction. Without systematic instruction, students may not have had an understanding of the task or how to improve their performance from one prompt to the next. Moreover, by not providing explicit instruction, results of the studies may be difficult to replicate. Finally, eight of the 17 studies (47%) involved journal writing, despite research indicating mathematics teachers were less likely to incorporate journal writing in their instruction over other types of mathematics writing (Gillespie et al., 2015; Swinson, 1992). Thus, there is a need to develop systematic and explicit interventions teachers would be more likely to utilize.

Several areas show promise for the development of more explicit and systematic intervention. Of the effective interventions, many studies provided explicit instruction on mathematics writing, including modeling the use of talk frames (Cohen et al., 2015); using routines to develop written mathematical arguments (Cross, 2009); providing instruction on paraphrasing word problems (Moran et al., 2014); coaching students to monitor mathematics writing using organization and meta-cognitive strategies (Glogger et al., 2012); and providing examples, modeling, and mini-lessons (Kostos & Shin, 2010). Requiring a written explanation about the solution to a particular problem also emerged as a trend among studies demonstrating positive results. Examples include requiring students to explain steps used in solving a problem (Idris, 2009; Tan & Garces-Bacsal, 2013), responses that required further explanation (Jurdak & Abu Zein, 1998; Kostos & Shin, 2010), and explanations on concepts learned in mathematics class (Lim & Pugalee, 2004). Providing students with verbal or written feedback from a peer (Fortescue, 1994) or teacher (Cross, 2009; Pugalee, 2004) was a final practice that emerged from studies of mathematics-writing interventions, although the practice was not examined experimentally.

**Improve the quality of empirical studies to identify and improve effective mathematics-writing interventions and instructional practices.** As previously stated, the quality of the empirical studies of intervention research in this synthesis was relatively poor. Although the purpose of this synthesis was not to provide a guide for implementing high-quality research, there were many gaps in this literature that should be addressed. To improve the quality of intervention research, future researchers should employ experimental, quasi-experimental, or single-subject designs. To the extent possible, future research should include and describe comparison groups, randomly assign students or classrooms to condition, analyze the data at the level that matches the unit of assignment, include measures of fidelity, include measures with high internal consistency, provide means and standard deviations for all conditions, and carefully describe methods for replication.

Researchers should also provide stronger description of the intervention by describing the writing type, providing the frequency and duration of instructional practices, including descriptions of teacher modeling, and stating how feedback is provided. Researchers should also include the length of intervention, amount of guided and independent practice opportunities, and amount of training required of teachers to implement the intervention with fidelity.

**Incorporate student measures of self-efficacy and motivation for mathematics writing into studies of interventions and assessments.** Our final recommendation concerns the surveys included in this synthesis. The surveys provided valuable pictures of different aspects of mathematics writing, including teacher instructional practices, student motivation, student self-efficacy, and student beliefs about the effectiveness of mathematics writing to improve their performance. In particular, the surveys examined in this synthesis examined the face validity of mathematics-writing practices. While this may have been important for justifying the early use of the mathematics-writing interventions and assessments, the measurement of these factors could be used in a more sophisticated way to improve understanding of the role factors play in assessment and intervention. That is, many of these factors may be mediators or moderators of the influence of mathematics-writing intervention on student mathematics-writing performance. Future research should incorporate measures of these factors within studies of mathematics-writing to determine their influence on these practices.

### 4.5 Recommendations for Teachers

Based on the scarcity of research, the variability of the empirical questions and research methodologies used, and the quality of the study reports, there are few recommendations we can provide for teachers. Because mathematics writing is expected in state standards and Common Core assessments used in the United States, however, teachers need to incorporate mathematics writing into their classrooms. Therefore, we provide a few recommendations based on the findings of this synthesis that teachers should implement cautiously. First, teachers should implement systematic and explicit instruction in mathematics writing for mathematics topics appropriate for the teacher's grade level. Second, when interpreting mathematics writing, teachers should try to determine whether the mathematics writing provides an accurate representation of students' mathematical knowledge. That is, teachers should determine whether difficulty with mathematics writing is due to a lack of general writing skill, weak mathematical skills, or both. Third, teachers should only use mathematics writing as a measure of students' ability to communicate in mathematics in a balanced approach to teaching mathematical skills and concepts. In other words, mathematics writing should not replace other measures of mathematical knowledge, and teachers should also teach students to communicate in mathematics orally.

### 4.6 Limitations

We centered our recommendations on providing guidance for improving the research on mathematics-writing assessments and interventions. We discussed many of the limitations of the research in this context. There are also limitations, however, to the methodology we used in this synthesis. First, we included only empirical research, including qualitative research only when authors provided some form of empirical data. This decision was made mainly for manageability purposes but may limit some of our findings. Second, we did not include dissertation data. Much of the empirical work involving dissertations was well-covered in the meta-analysis by Bangert-Drowns and colleagues (2004), so we decided to limit our findings to peer-reviewed publications. Third, we included research published in 1990 or later. Although we made this decision to align with the first year that the NCTM described communication as an essential practice for mathematics classrooms, we recognize that we may have missed important work in mathematics writing that may have occurred prior to this date, and these dates may not have the same relevance to international use of mathematics writing.

### 4.7 Conclusion

The studies included in this synthesis indicate that teachers can use mathematics writing to help students communicate in mathematics and to learn mathematics. All students should have opportunities to participate in mathematical pre-writing, explanatory writing, argumentative writing, and mathematically creative writing (Casa et al., 2016), and teachers need to provide mathematics-writing opportunities. As high-stakes assessments use mathematics writing to gauge mathematical competency, it is necessary to conduct high-quality research to determine the most efficacious assessment and intervention practices. Such research is necessary to provide better direction and recommendations for teachers required to implement these practices.

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